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Threshold corrections to m_b and the $b\bar{b} \to H_i^0$ production in CP-violating SUSY scenarios

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Abstract

The inclusion of supersymmetric threshold corrections to the b-quark mass has dramatic consequences in scenarios with large CP-mixing effects in the Higgs sector. In particular, when the phase of the combination $M_{\tilde{g}}\mu$ is $\sim 180^{\rm o} \pm 30^{\rm o}$, the lightest sbottom squark becomes tachyonic and, possibly, the b-quark Yukawa coupling nonperturbative for values of $\tan\beta$ ranging from intermediate up to large or very large, depending on the size of $arg(A_t\mu)$, $arg(A_b\mu)$, and the details of the spectrum. For these scenarios, when allowed, as well as scenarios with different values of $arg(M_{\tilde{g}}\mu)$, the cross sections for the production of the three neutral Higgs bosons through b-quark fusion have interesting dependences on $arg(A_t\mu)$ and $arg(A_b\mu)$, and the deviations induced by the m_b corrections are rather large. In general, such production channel cannot be neglected with respect to the production through gluon fusion. For large CP-mixing effects, the lightest neutral Higgs boson can be mainly CP odd and the b-quark fusion becomes its main production mechanism. Searches at the Tevatron and the LHC can easily detect such a Higgs boson, or constrain the CP-violating scenarios that allow it.

It is well known that threshold corrections to the b-quark mass due to the virtual exchange of supersymmetric particles can be large and do not decouple in the limit of heavy superpartners [1]. They turn out to be quite substantial [2], also for moderate values of $\tan\beta$, in scenarios that maximize the CP-mixing effects induced in the Higgs sector at the quantum level by phases in the supersymmetric and soft supersymmetry-breaking mass parameters (such scenarios were first studied in Refs. [3, 4]).

These corrections affect the vertex $\bar{b}$-b-H_i^0, which is responsible for one of the production mechanisms of neutral Higgs bosons at hadron colliders. If no CP-mixing effects are present in the Higgs sector, H_i^0 is either one of the two CP-even states h and H, or the CP-odd state A [5]. In this case, the mechanism of b-quark fusion has relevance, for example, for the production of A [6] (and in the decoupling limit, also in the case of the production of H). Indeed, a factor $\tan\beta$ in the $\bar{b}$-b-A coupling can give a production rate comparable to, or even dominant over, the rate of production through gluon fusion. On the contrary, lacking this $\tan\beta$ factor, the b-quark fusion plays a minor role in the production of the light CP-even Higgs boson with respect to the gluon-fusion, which, although loop-induced, is advantaged by the large yield of gluons in the proton. In scenarios with a CP-violating Higgs sector, the three mass eigenstates H_i^0 are mixed states with both CP-even and CP-odd components *. Thus, it is conceivable that, if the CP-violating mixing effects are large, the production through b-quark fusion may be relevant for all mass eigenstates of neutral Higgs bosons.

The aim of this paper is to investigate this issue, while consistently including the supersymmetric threshold corrections to the b-quark mass. We find that these corrections can have a strong impact if phases are present in supersymmetric and supersymmetry-breaking parameters, in that they can exclude intermediate/large values of $\tan\beta$, leaving nevertheless allowed the very large ones. Predictably, when these corrections and the CP-mixing effects in the Higgs sector are large, the production cross sections through b-quark fusion for all the neutral Higgs bosons are affected, with that for the lightest one, possibly, dramatically enhanced. The effect of the supersymmetric threshold corrections to m_b on these cross sections may remain rather large also when some phases in these scenarios acquire trivial values, i.e. 0° or 180°.

We illustrate our findings for scenarios that have been dubbed CPX scenarios [10], and that tend to maximize the CP-mixing effects in the Higgs sector †. These scenarios are identified by the spectrum:

$$|A_t| = |A_b| = 2c_A\, M_{\rm SUSY}\,, \quad |\mu| = 4c_\mu\, M_{\rm SUSY}\,, \quad m_{\widetilde{Q}_3,\widetilde{U}_3,\widetilde{D}_3} = M_{\rm SUSY}\,, \quad |M_{\widetilde{g}}| = 1\,{\rm TeV}\,, \quad (1)$$

where A_t and A_b are the complex trilinear soft terms for the third generation squarks, μ is the complex supersymmetric Higgs(ino) mass parameter; $m_{\widetilde{Q}_3}$, $m_{\widetilde{U}_3}$ and $m_{\widetilde{D}_3}$ are the real

*Phenomenological consequences of these mixings can be found in Refs. [7, 8, 9].

†In general, large CP-violating phases in supersymmetric models are indirectly forbidden by the non-observation of electron and neutron electric dipole moments. These constraints, however, can be evaded by cancellations between the one- and higher-loop contributions [11] to the electric dipole moments if the first two generations of sfermions are heavier than $\mathcal{O}(1\,{\rm TeV})$ [12, 13, 14]. Although somewhat tuned, these scenarios have sparked quite some interest, and need to be probed directly through collider searches of CP violation in the Higgs sector.

soft-breaking masses for the third generation squarks; $M_{\widetilde{g}}$ is the complex gluino mass, and c_A, c_μ are real numbers. (The mass parameters entering the slepton sector and the mass of the two weak gauginos are irrelevant for our discussion.) The remaining parameter needed to specify the Higgs sector is chosen here to be the pole mass of the charged Higgs boson, $m_{H^\pm}$ ‡. Strictly speaking the spectrum of CPX scenarios has $c_A = c_\mu = 1$. Here, we extract informations also for cases with $c_A, c_\mu < 1$. We take $M_{\rm SUSY} = 0.5\,{\rm TeV}$; vary $\tan\beta$, the ratio of the *v.e.v.*'s v_d and v_u acquired by H_d^0 and H_u^0 at the minimum of the Higgs potential, and vary also $m_{H^\pm}$. As for the phases [15] present in these scenarios, we take as free parameters the arguments of the products $A_t\mu$, $A_b\mu$, which we assume to be equal, and of the product $M_{\tilde{g}}\mu$, respectively $\Phi_{A\mu} \equiv \arg(A_t\mu) = \arg(A_b\mu)$, and $\Phi_{g\mu} \equiv \arg(M_{\tilde{g}}\mu)$. When relevant, we discuss also the *four* CP-conserving cases obtained for $\Phi_{A\mu}, \Phi_{g\mu} = 0^{\rm o}, 180^{\rm o}$. Our numerical analyses make use of the recently-developed program CPsuperH [16].

We start observing that, once the threshold corrections to the b-quark mass are included, the in-general-complex b-quark Yukawa coupling is

$$h_b = \frac{\sqrt{2}\, m_b}{v\cos\beta}\,\frac{1}{R_b}\,, \tag{2}$$

where $v^2 = v_d^2 + v_u^2$, with $v \simeq 254\,{\rm GeV}$, and $v_d = v\cos\beta$, $v_u = v\sin\beta$. The factor R_b:

$$R_b = 1 + \kappa_b \tan\beta \tag{3}$$

collects in κ_b the finite corrections § to the b-quark mass. In turn, κ_b can be split as

$$\kappa_b = \epsilon_g + \epsilon_H\,, \tag{4}$$

where ϵ_g and ϵ_H are, respectively, the contribution coming from the sbottom-gluino exchange diagram and from the stop-Higgsino diagram. Their explicit form is

$$\epsilon_g = \frac{2\alpha_s}{3\pi}\, M_{\widetilde{g}}^* \mu^* I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, |M_{\widetilde{g}}|^2)\,, \qquad \epsilon_H = \frac{|h_t|^2}{16\pi^2}\, A_t^* \mu^* I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, |\mu|^2)\,. \tag{5}$$

The one-loop function $I(a,b,c)$ is well known and can be found, for example, in [17]. The left-right mixing elements in the matrices for the sbottom- and stop-mass squared used to obtain these corrections, are, in our convention,

$$\frac{1}{\sqrt{2}}\, h_b^*(A_b^* v_d - \mu v_u)\,, \qquad \frac{1}{\sqrt{2}}\, h_t^*(A_t^* v_u - \mu v_d)\,. \tag{6}$$

For the spectrum of Eq. (1) with $c_A = c_\mu = 1$ and $M_{\rm SUSY} = 0.5\,{\rm TeV}$, it is $|\mu|^2, |M_{\widetilde{g}}|^2 \gg M_{\rm SUSY}^2$ and, as a consequence, the sbottom-gluino corrections are, in absolute value, much larger than those obtained from stop-Higgsino exchange. For α_s and h_t at the scale $M_{\rm SUSY}$, i.e.

‡We remind that the issue of electroweak-symmetry breaking, possibly radiatively induced as in the constrained Minimal Supersymmetric Standard Model is not addressed in these scenarios.

§There are additional corrections $\delta h_b/h_b$ to R_b which are not enhanced by $\tan\beta$. We neglect them in our discussion but their effect is fully included in the numerical analysis [16].

$\alpha_s \sim 0.1$ and $h_t \sim 1$, $|\epsilon_g|$ has a value ~ 0.05 and it is about one order of magnitude larger than $|\epsilon_H|$ ($|\epsilon_g|$ is larger than $|\epsilon_H|$ by the factor $\sim \pi\, |\mu|^2/|A_t M_{\widetilde{g}}|$) ¶. In this case, the threshold corrections to the b-quark Yukawa coupling are strongly affected by the phase $\Phi_{g\mu}$, whereas the dependence on $\Phi_{A\mu}$ is weak. We remind that, at the one-loop level, the corrections inducing the CP-mixing in the Higgs sector are sensitive only to $\Phi_{A\mu}$. $\Phi_{g\mu}$ affects this mixing at the two-loop level through the one-loop corrections to the b- and t-quark masses [4]. Notice that the hierarchy $|\epsilon_g| \gg |\epsilon_H|$ holds in general, except in the somewhat unlikely cases $|M_{\widetilde{g}}| \ll |A_t|$ or $|\mu^2|\,, M^2_{\rm SUSY} \ll |A_t M_{\widetilde{g}}|$. Possible variations of the renormalization scale of α_s and h_t do not affect this statement, neither have any substantial impact on the numerical results for the production cross sections of the various states H_i.

The effect of the radiative corrections becomes most significant for $\Phi_{g\mu} = \Phi_{A\mu} = 180^{\rm o}$, when dramatic constraints on $\tan\beta$ can be obtained. Indeed, to prevent the lighter sbottom squark $\tilde{b}_1$ from becoming tachyonic, the b-quark Yukawa coupling is constrained as

$$|h_b| \;\lesssim\; \frac{\sqrt{2}\, M^2_{\rm SUSY}}{v\,|\mu|} \left[1 + \mathcal{O}\!\left(\frac{|A_b|}{|\mu| \tan\beta} \right) \right] , \tag{7}$$

where we have taken $v_u \sim v$ and neglected the m_b^2- and D-term contributions to the diagonal elements of the matrix for the sbottom-mass squared. Since $\mathcal{O}(|A_b|/|\mu|\tan\beta)$ is in general negligible, the constraint on h_b can be easily recast into a constraint on $\tan\beta$. That is, the region of $\tan\beta$:

$$\frac{1}{|\kappa_b| + \dfrac{m_b|\mu|}{M^2_{\rm SUSY}}} \;\lesssim\; \tan\beta \;\lesssim\; \frac{1}{|\kappa_b| - \dfrac{m_b|\mu|}{M^2_{\rm SUSY}}} \tag{8}$$

is *not* allowed when

$$|\kappa_b| > \frac{m_b|\mu|}{M^2_{\rm SUSY}} \,, \qquad\qquad \kappa_b = -|\kappa_b| \,. \tag{9}$$

These conditions are easily satisfied by the spectrum in Eq. (1), for any value of c_A and c_μ.

The constraint of a nontachyonic $\tilde{b}_1$ excludes the $\tan\beta$ region $12 \lesssim \tan\beta \lesssim 30$ for the scenario in Eq. (1) with $M_{\rm SUSY} = 0.5\,{\rm TeV}$, $c_A = c_\mu = 1$, and $\Phi_{g\mu} = \Phi_{A\mu} = 180^{\rm o}$. The value $m_b = 3\,{\rm GeV}$ was used here and will be used also for all other numerical evaluations throughout this paper. In such a case, the coupling h_b has at most the value 0.7 in the allowed region, as shown by the left frame of Fig. 1. By imposing that the lightest-Higgs boson mass exceeds, say, $115\,{\rm GeV}$ ‖, the excluded region gets extended up to $\tan\beta \sim 50$. Notice that the limiting case $\tan\beta \to \infty$, i.e. $v_d \to 0$, is not a problematic one, when it comes to obtain acceptable values of $m_{\tilde{b}_1}$ and m_{H_1}. The h_b coupling is, in this case, $h_b \sim \sqrt{2} m_b / v\kappa_b$

¶Such large values of $|\epsilon_g|$ and $|\epsilon_H|$ are specific to the scenarios considered here and are compatible with those found in Ref. [18].

‖This value is reminiscent of the lower bound on the lightest-Higgs boson from LEP2 [19]. This was, however, obtained in the CP-conserving case and in the limit of a heavy CP-odd Higgs boson. Moreover, it is known [10] that in CP-violating scenarios the lower bound that can be deduced from LEP data may be considerably weaker than that reported in Ref. [19]. In our case case, the choice of this value is motivated by the fact that it allows simple comparisons with existing results in the literature. Particularly important will be later on the comparison with results presented in Ref. [20], where the same choice is made.

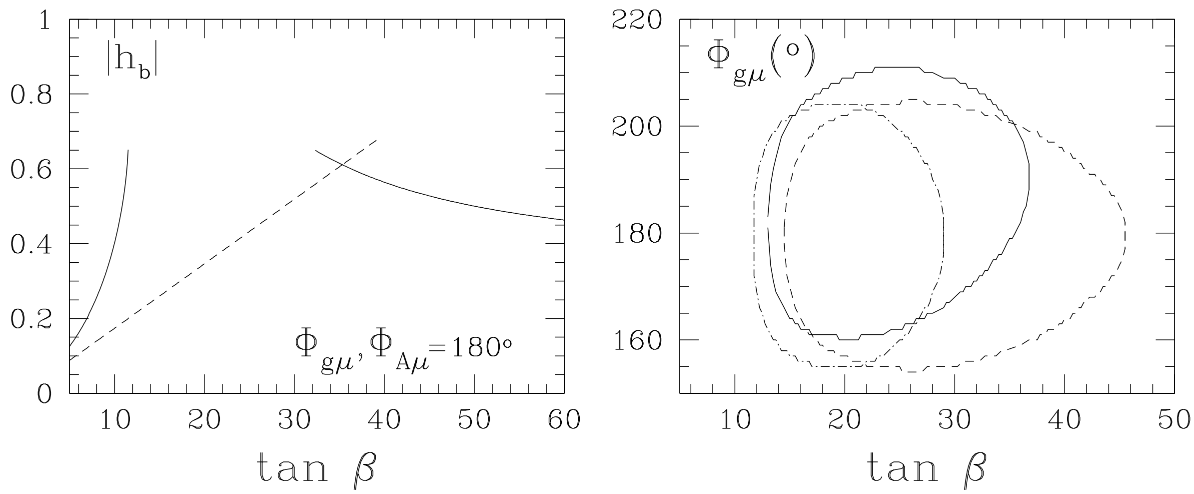

Figure 1: *Absolute value of the Yukawa coupling of the b quark, vs.* $\tan\beta$ *(left frame) for the scenario of Eq. 1, with* $c_A = c_\mu = 1$*,* $M_{\rm SUSY} = 0.5\,TeV$*, and* $\Phi_{A\mu} = \Phi_{g\mu} = 180^o$*. For the same scenario, but fixed values of* $\Phi_{A\mu}$*, the regions of the plane* $(\Phi_{g\mu}, \tan\beta)$ *in which the mass of the* $\tilde{b}_1$ *squark is vanishing are shown on the right frame. They are enclosed by a dashed, solid, dot-dashed line corresponding to* $\Phi_{A\mu} = 0^o, 90^o, 180^o$*.*

and the mass of the b quark is generated at the quantum level [17]. For smaller values of c_μ, for example such that $|\mu| = M_{\rm SUSY}$, the excluded region of $\tan\beta$ at intermediate/large values is further enlarged by the fact that $|h_b|$ grows rapidly above perturbative levels.

This $\tan\beta$ exclusion occurs also for nontrivial values of $\Phi_{g\mu}$ and $\Phi_{A\mu}$, as it can be seen in the right frame of Fig. 1. In it, we show explicitly the regions of the plane $\Phi_{g\mu}$-$\tan\beta$ in which the mass of the $\tilde{b}_1$ squark is negative. They are enclosed by a dashed, solid, dot-dashed line, corresponding to $\Phi_{A\mu} = 0^o, 90^o, 180^o$, respectively. The scenario chosen for this frame is, again, that of Eq. (1) with $M_{\rm SUSY} = 0.5\,{\rm TeV}$ and $c_A = c_\mu = 1$. Regions of $\tan\beta$ to be excluded are found for $\Phi_{g\mu} \sim 180^o \pm 30^o$. This pattern has to be compared with the very large $\tan\beta$ exclusion typical of constrained supersymmetric models with $\Phi_{A\mu} = \Phi_{g\mu} = 0^o$, in which the threshold corrections to m_b are not taken into account [21].

We turn now to consider the b-b-H_i couplings. Once the threshold corrections to the b-quark mass are included, the effective Lagrangian for the interaction of the neutral Higgs boson to b quarks can be written as

$$\mathcal{L} \ = \ -\frac{m_b}{v}\,\bar{b}\left(g_\phi^S + i g_\phi^P \gamma_5\right) b\,\phi\,, \tag{10}$$

with $\phi = (\phi_1, \phi_2, a)$, where ϕ_1 and ϕ_2 are the CP-even parts of the neutral components of the two doublets:

$$H_d^0 = \frac{1}{\sqrt{2}}\left(v_d + \phi_1 + i a_1\right)\,, \qquad H_u^0 = \frac{1}{\sqrt{2}}\left(v_u + \phi_2 + i a_2\right)\,, \tag{11}$$

and a is a combination of the two CP-odd components a_1 and a_2, $a = -a_1 \sin\beta + a_2 \cos\beta$. The orthogonal combination, $G^0 = a_1 \cos\beta + a_2 \sin\beta$, is the Goldstone mode. The couplings

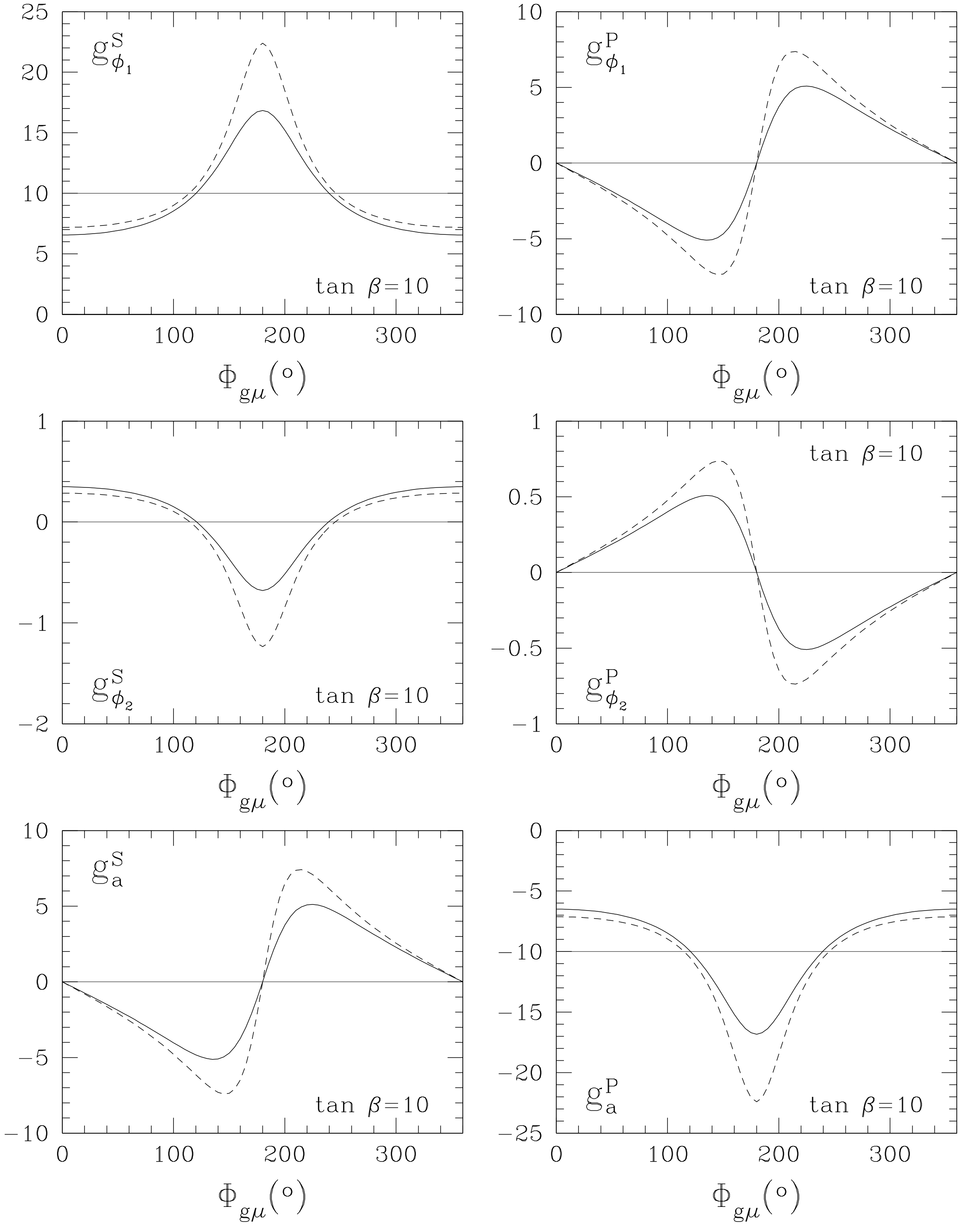

Figure 2: *Couplings $g_\phi^{S,P}$ vs. $\Phi_{g\mu}$, for the spectrum in Eq. (1) with $c_A = c_\mu = 1$, $M_{\rm SUSY} = 0.5$ TeV, and $\tan\beta = 10$. The solid lines are for $\Phi_{A\mu} = 0^{\rm o}$, the dashed ones for $\Phi_{A\mu} = 180^{\rm o}$. The horizontal lines indicate the values of the uncorrected couplings.*

$g_\phi^{S,P}$ are [13]

$$
\begin{aligned}
g^S_{\phi_1} &= \frac{1}{\cos\beta}\,\Re\mathrm{e}\left(\frac{1}{R_b}\right), & g^P_{\phi_1} &= \frac{\tan\beta}{\cos\beta}\,\Im\mathrm{m}\left(\frac{\kappa_b}{R_b}\right), \\
g^S_{\phi_2} &= \frac{1}{\cos\beta}\,\Re\mathrm{e}\left(\frac{\kappa_b}{R_b}\right), & g^P_{\phi_2} &= -\frac{1}{\cos\beta}\,\Im\mathrm{m}\left(\frac{\kappa_b}{R_b}\right), \\
g^S_a &= (\tan^2\beta+1)\,\Im\mathrm{m}\left(\frac{\kappa_b}{R_b}\right), & g^P_a &= -\Re\mathrm{e}\left(\frac{\tan\beta-\kappa_b}{R_b}\right),
\end{aligned}
\tag{12}
$$

and for values of $\tan\beta$ such that $|\kappa_b|\tan\beta \sim 1$ (with one of the two, or both possibilities: $\Re\mathrm{e}(\kappa_b)\tan\beta \sim 1$, $\Im\mathrm{m}(\kappa_b)\tan\beta \sim 1$) reduce to

$$
\begin{aligned}
g^S_{\phi_1} &= \frac{\tan\beta}{|R_b|^2}\left[1+\Re\mathrm{e}(\kappa_b)\tan\beta\right], & g^P_{\phi_1} &= \frac{\tan\beta}{|R_b|^2}\left[\Im\mathrm{m}(\kappa_b)\tan\beta\right], \\
g^S_{\phi_2} &= \frac{1}{|R_b|^2}\left[\Re\mathrm{e}(\kappa_b)\tan\beta+|\kappa_b|^2\tan^2\beta\right], & g^P_{\phi_2} &= -\frac{1}{|R_b|^2}\left[\Im\mathrm{m}(\kappa_b)\tan\beta\right], \\
g^S_a &= \frac{\tan\beta}{|R_b|^2}\left[\Im\mathrm{m}(\kappa_b)\tan\beta\right], & g^P_a &= -\frac{\tan\beta}{|R_b|^2}\left[1+\Re\mathrm{e}(\kappa_b)\tan\beta\right].
\end{aligned}
\tag{13}
$$

Before proceeding further, we list in the following some of the interesting features of these couplings. We illustrate them numerically for a specific CPX spectrum, i.e. with $c_A = c_\mu = 1$, $M_{\rm SUSY} = 0.5\,$TeV, and $\tan\beta = 10$, which is a value not plagued by the problem of a tachyonic $\tilde{b}_1$ squark discussed earlier. For visual clarity we also illustrate some of these features in Fig. 2.

• If no threshold corrections are included, the only nonvanishing couplings are $g^S_{\phi_1} = 1/\cos\beta$ and $g^P_a = -\tan\beta$. The inclusion of these corrections affects these two couplings mainly through the factor $\Re\mathrm{e}(1/R_b)$, which is a suppression or an enhancement factor, depending on the value of $\Phi_{g\mu}$, and varies between $\sim 1/(1+|\epsilon_g|\tan\beta)$ and $\sim 1/(1-|\epsilon_g|\tan\beta)$, obtained for $\Phi_{g\mu} = 0^{\rm o}$ and $\Phi_{g\mu} = 180^{\rm o}$, respectively. We have used here the fact that $|\epsilon_g| \gg |\epsilon_H|$, which, as previously discussed, is rather generic in our scenarios. Notice also that the factor $\Re\mathrm{e}(1/R_b)$ is larger than 1 for $\cos\Phi_{g\mu} \lesssim -|\epsilon_g|\tan\beta$. For $\tan\beta = 10$ and the spectrum specified above, this happens when $135^{\rm o} \lesssim \Phi_{g\mu} \lesssim 225^{\rm o}$, and $|g^S_{\phi_1}|$ and $|g^P_a|$ reach the maximum of 15 or ~ 20, depending on the value of $\Phi_{A\mu}$ (i.e. $0^{\rm o}$ or $180^{\rm o}$), at $\Phi_{g\mu} = 180^{\rm o}$.

• The inclusion of threshold corrections is responsible for the appearance of the other four couplings: $g^S_{\phi_2}$ and $g^P_{\phi_2}$, the smallish ones, *i.e.* without the overall factor of $\tan\beta$ that $g^S_{\phi_1}$, g^P_a have (when $|\kappa_b|\tan\beta \sim 1$), $g^P_{\phi_1}$ and g^S_a, the large ones, with the $\tan\beta$ factor.

• If no CP phases are present, only $g^S_{\phi_1}$, $g^S_{\phi_2}$, and g^P_a are nonvanishing and, again, the effect of the threshold corrections is more significant for $\Phi_{g\mu} = 180^{\rm o}$ than for $\Phi_{g\mu} = 0^{\rm o}$.

• As the phase $\Phi_{g\mu}$ varies, the moduli of the couplings $g^P_{\phi_1}$, $g^P_{\phi_2}$, and g^S_a reach their extremal values at $\cos\Phi_{g\mu} \approx -2|\epsilon_g|\tan\beta/(1+|\epsilon_g|^2\tan^2\beta)$. In the specific case considered here, this is 5 or 8 for $|g^P_{\phi_1}|$ and $|g^S_a|$ at $\Phi_{g\mu} \approx 150^{\rm o}$ and $210^{\rm o}$, depending on the value of $\Phi_{A\mu}$. In contrast, $|g^P_{\phi_2}|$, as well as $|g^S_{\phi_2}|$, never exceed 1.2.

The states ϕ_1, ϕ_2, and a are not yet the neutral Higgs boson mass eigenstates. Their real and symmetric 3×3 matrix $\mathcal{M}^2_H$ has nonvanishing entries that mix the two states ϕ_1 and ϕ_2

as well as nonvanishing CP-violating entries that mix a with ϕ_1 and ϕ_2 and are proportional to $\Phi_{A\mu}$. The diagonalization of this matrix through an orthogonal matrix O,

$$O^T \mathcal{M}_H^2 \, O = \mathsf{diag}\,(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2)\,, \tag{14}$$

yields the three eigenstates H_1, H_2, H_3, ordered for increasing value of their masses. In the limit of vanishing $\Phi_{A\mu}$, H_1 is h, H_2 and H_3 are H and A, or vice versa, A and H, depending on the values of m_A and m_H. After this rotation, the effective Lagrangian of Eq. (10) becomes

$$\mathcal{L} \;\rightarrow\; -\frac{m_b}{v}\,\bar{b}\left(g_{H_i}^S + i g_{H_i}^P \gamma_5\right) b\, H_i\,, \tag{15}$$

where $g_{H_i}^S$ and $g_{H_i}^P$ are

$$g_{H_i}^S = O_{\alpha i}\, g_\alpha^S\,, \qquad g_{H_i}^P = O_{\alpha i}\, g_\alpha^P\,, \tag{16}$$

with the index α running over (ϕ_1, ϕ_2, a) and i over $(1,2,3)$.

We are now in a position to evaluate the production cross sections of the neutral Higgs bosons H_i via b-quark fusion at hadron colliders. They can be expressed as:

$$\sigma(\mathrm{had}_1\mathrm{had}_2 \to b\bar{b} \to H_i) = \sigma(b\bar{b} \to H_i) \int_{\tau_i}^1 \mathrm{d}x \left[\frac{\tau_i}{x}\, b_{\mathrm{had}_1}(x,Q)\, \bar{b}_{\mathrm{had}_2}\!\left(\frac{\tau_i}{x}, Q\right) + (b \leftrightarrow \bar{b})\right]\,, \tag{17}$$

where $b_{\mathrm{had}_i}(x,Q)$ and $\bar{b}_{\mathrm{had}_i}(x,Q)$ are the b- and $\bar{b}$-quark distribution functions in the hadron had_i, Q is the factorization scale, and τ_i the Drell–Yan variable $\tau_i = m_{H_i}^2/s$, with s the invariant hadron-collider energy squared. Finally, the partonic cross section is

$$\sigma(b\bar{b} \to H_i) = \frac{m_b^2}{v^2}\frac{\pi}{6 m_{H_i}^2}\left[(g_{H_i}^S)^2 + (g_{H_i}^P)^2\right]\,. \tag{18}$$

The sum of the couplings squared

$$\left[(g_{H_i}^S)^2 + (g_{H_i}^P)^2\right] = \left[\left(O_{ai} g_a^S + O_{\phi_1 i} g_{\phi_1}^S + O_{\phi_2 i} g_{\phi_2}^S\right)^2 + \left(O_{ai} g_a^P + O_{\phi_1 i} g_{\phi_1}^P + O_{\phi_2 i} g_{\phi_2}^P\right)^2\right] \tag{19}$$

reduces, in the limit in which no threshold corrections are included, to

$$\left[(g_{H_i}^S)^2 + (g_{H_i}^P)^2\right]|_{no-thresh-corr} = \left[O_{\phi_1 i}^2 (g_{\phi_1}^S)^2 + O_{ai}^2 (g_a^P)^2\right]\,, \tag{20}$$

with $g_{\phi_1}^S = 1/\cos\beta$ and $g_a^P = -\tan\beta$. For $i=1$, in the limit of large $m_{H^\pm}$, $O_{\phi_1 1} \to \cos\beta$ and $O_{a1} \to 0$, leading to the usual Standard Model coupling of the lightest neutral Higgs boson to b quarks, in both, CP-conserving and CP-violating scenarios. That is, the overall $\tan\beta$ dependence of the coupling $g_{\phi_1}^S$ is killed by the mixing element $O_{\phi_1 1}$.

Through the inclusion of the threshold corrections to the b quark in CP-conserving scenarios, the element $g_{\phi_2}^S$ gets switched on. Being one of the small couplings, i.e. one without an overall $\tan\beta$ factor when $\tan\beta$ is large, $g_{\phi_2}^S$ is not expected to produce great numerical deviations in the values of the cross sections at large $\tan\beta$. Deviations of $O(1)$ can only be induced by the factors $1/|R_b|^4$, which enhance or suppress the uncorrected cross sections,

depending on the sign of the threshold corrections. It is however in CP-violating scenarios, when two couplings with an overall $\tan\beta$ dependence are switched on, $g^P_{\phi_1}$ and g^S_a, that the pattern of Higgs production through b-quark fusion can become very different. This, of course, if the projection of Higgs boson current-eigenstates to mass-eigenstates is not particularly destructive.

To analyze the behaviour of the sum in Eq. (19), we use the approximation:

$$\left[(g^S_{H_i})^2 + (g^P_{H_i})^2\right] \approx \left[O^2_{\phi_1 i} + O^2_{ai}\right]\left[(g^S_a)^2 + (g^P_a)^2\right] . \tag{21}$$

In this, we have parametrized the sum $[(g^S_{H_i})^2 + (g^P_{H_i})^2]$ in terms of the only elements $O_{\phi_1 i}$ and O_{ai} that appear in Eq. (20), i.e. in the case in which no threshold corrections to the b-quark mass are included. This approximation relies on the properties of the couplings $g^{S,P}_\phi$ discussed above, i.e. on the fact that $g^S_{\phi_1} \approx -g^P_a$, $g^P_{\phi_1} \approx g^S_a$, and that $g^S_{\phi_2}$ and $g^P_{\phi_2}$ can be neglected. In the scenarios considered here, this approximation turns out to be valid for most of the relevant values of the phases $\Phi_{g\mu}$ and $\Phi_{A\mu}$.

To strengthen our point, we show explicitly the mixing elements $O_{\phi i}$ for the scenarios already considered in Fig. 2. We show them in Fig. 3 versus $\Phi_{A\mu}$, but with $\Phi_{g\mu}$ fixed at 180^o. As already remarked, the elements $O_{\phi i}$ have a rather weak dependence on $\Phi_{g\mu}$, coming from the two-loop corrections to the Higgs potential. The charged Higgs boson mass is solved in order to have $m_{H_1} = 115\,\mathrm{GeV}$, for all values of $\Phi_{A\mu}$ and $\Phi_{g\mu}$, and it is therefore different for different values of these phases. As a consequence, also m_{H_2} and m_{H_3} are varying quantities. Notice that, for $\Phi_{A\mu} \approx 100^o$, H_1 is predominantly the CP-odd a boson, whereas H_2 and H_3 are mainly ϕ_2 and ϕ_1, respectively. For these values of $\Phi_{A\mu}$, the charged Higgs boson is relatively light, i.e. it has a mass $\lesssim 200\,\mathrm{GeV}$.

A comparison of Figs. 2 and 3 indicates that the approximation in Eq. (21) is probably not adequate when $[O^2_{\phi_1 i} + O^2_{ai}] \ll 1$, since the neglect of the elements $g^S_{\phi_2}$ and $g^P_{\phi_2}$ cannot be justified. Therefore, we keep the exact expression in Eq. (19) for the calculation of the production cross sections, but we use this approximation in order to clarify the $\tan\beta$ dependence of the sum $[(g^S_{H_i})^2 + (g^P_{H_i})^2]$. By substituting in Eq. (21) the expression of the couplings $g^{S,P}_\phi$ at large $\tan\beta$, we obtain:

$$\left[(g^S_{H_i})^2 + (g^P_{H_i})^2\right] \approx \left(O^2_{\phi_1 i} + O^2_{ai}\right)\frac{\tan^2\beta}{|R_b|^2} . \tag{22}$$

That is to say, the terms in $\tan^4\beta$ cancel identically. It remains only a factor of $\tan^2\beta$ suppressed or enhanced by $1/|R_b|^2$, in which all dependence on $\Phi_{g\mu}$ gets concentrated. Whether this factor of $\tan^2\beta$ is inherited unharmed by the cross sections for all Higgs bosons H_i depends on the matrix elements O_{ai} and $O_{\phi_1 i}$. For the values of $\Phi_{A\mu}$ for which the charged Higgs boson is not very heavy, and the three Higgs boson states are significantly mixed, we expect this to be the case, in general for all three states, and in particularly also for the lightest one. That is, for $\Phi_{A\mu} \approx 100^o$, we expect a large enhancement of the production cross section for H_1, but presumably a suppression of that for H_2, as an inspection of Fig. 3 suggests.

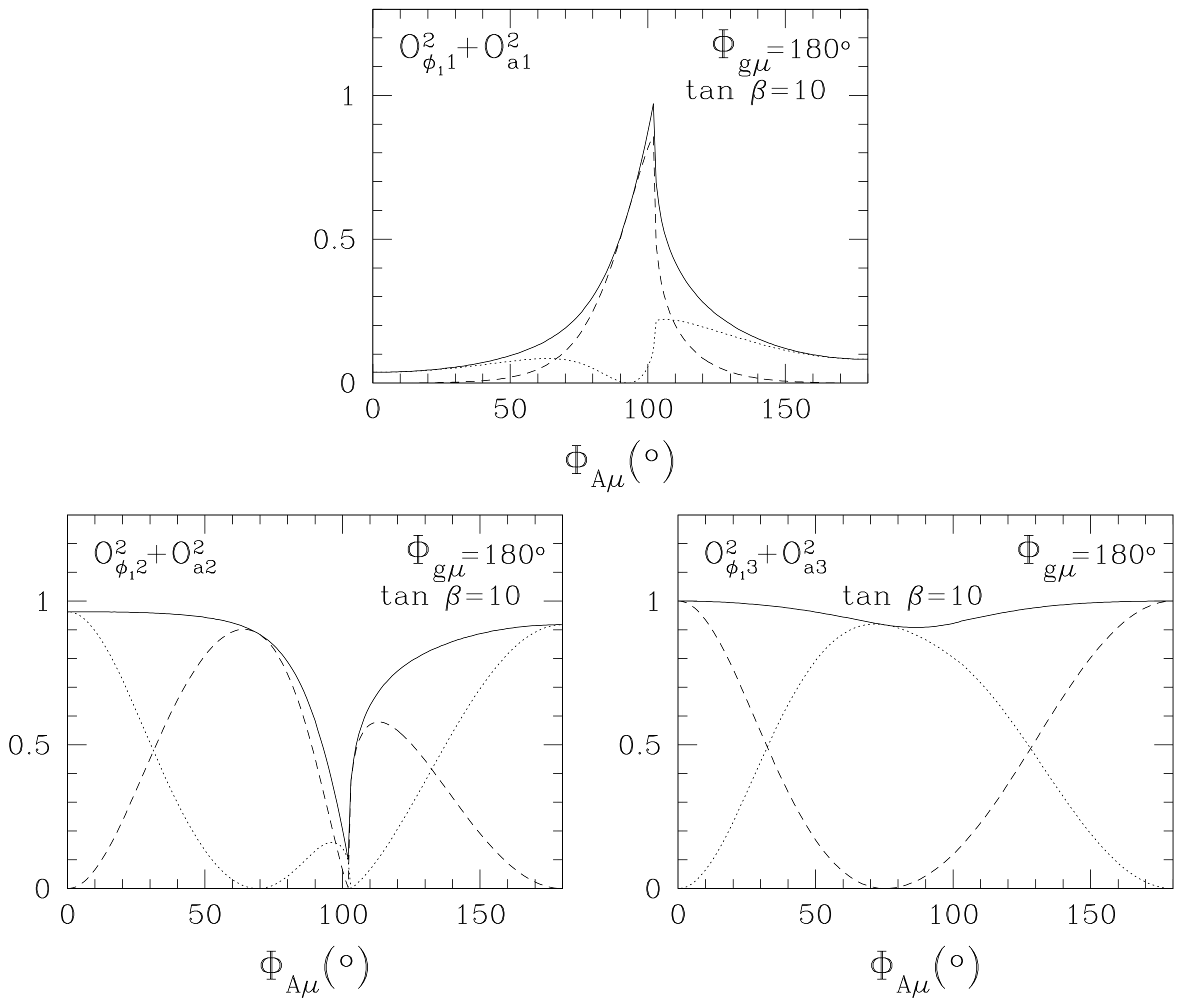

Figure 3: *The sums $O^2_{\phi_1 i} + O^2_{ai}$ vs. $\Phi_{A\mu}$, for the spectrum of Eq. (1) with $c_A = c_\mu = 1$, $\tan\beta = 10$, as in Fig. 2, and $m_{H_1} = 115$ GeV, $\Phi_{g\mu} = 180^o$. The dashed lines show O^2_{ai}; the dotted ones, $O^2_{\phi_1 i}$.*

We show the partonic cross sections in Fig. 4, as functions of $\Phi_{A\mu}$, for $\Phi_{g\mu} = 0^o$ (dashed lines) and 180^o (solid lines). The dotted lines indicate the cross sections without threshold corrections to the b-quark mass. The cross sections for all neutral Higgs bosons are shown for $\tan\beta = 10$, that for H_1 also for $\tan\beta = 5$. As mentioned, the two upper frames show cross sections for constant m_{H_1}, i.e. $m_{H_1} = 115\,\mathrm{GeV}$, which cannot actually be realized for all values of $\Phi_{A\mu}$ in the case of $\tan\beta = 5$ (m_{H_1} tends to be lighter for $\Phi_{A\mu} \lesssim 50^o$ and $\Phi_{A\mu} \gtrsim 130^o$). In contrast, in the two lower frames, m_{H_2} and m_{H_3} are different for different values of $\Phi_{A\mu}$: $m_{H_2}(m_{H_3})$ reaches the maximum of $\sim 240(250)\,\mathrm{GeV}$ at $\Phi_{A\mu} = 0^o$ and $\Phi_{g\mu} = 180^o$, the minimum of $\sim 120(150)\,\mathrm{GeV}$ for $\Phi_{A\mu} \sim 90^o$, nearly independently of $\Phi_{g\mu}$. All cross sections show the typical pattern already observed for $O^2_{\phi_1 i} + O^2_{ai}$ in Fig. 3, with modulations in the cases $i = 2, 3$, due to varying values of the corresponding Higgs masses. Notice the increase of almost two orders of magnitude in the cross section for H_1, at $\Phi_{A\mu} \approx 100^o$, when the value of $\tan\beta$ is only doubled. Indeed, for $i = 1$, the sum $O^2_{\phi_1 i} + O^2_{ai}$ is larger for $\tan\beta = 10$ than for $\tan\beta = 5$, when larger values of $m_{H^\pm}$ are needed to ensure that $m_{H_1} = 115\,\mathrm{GeV}$.

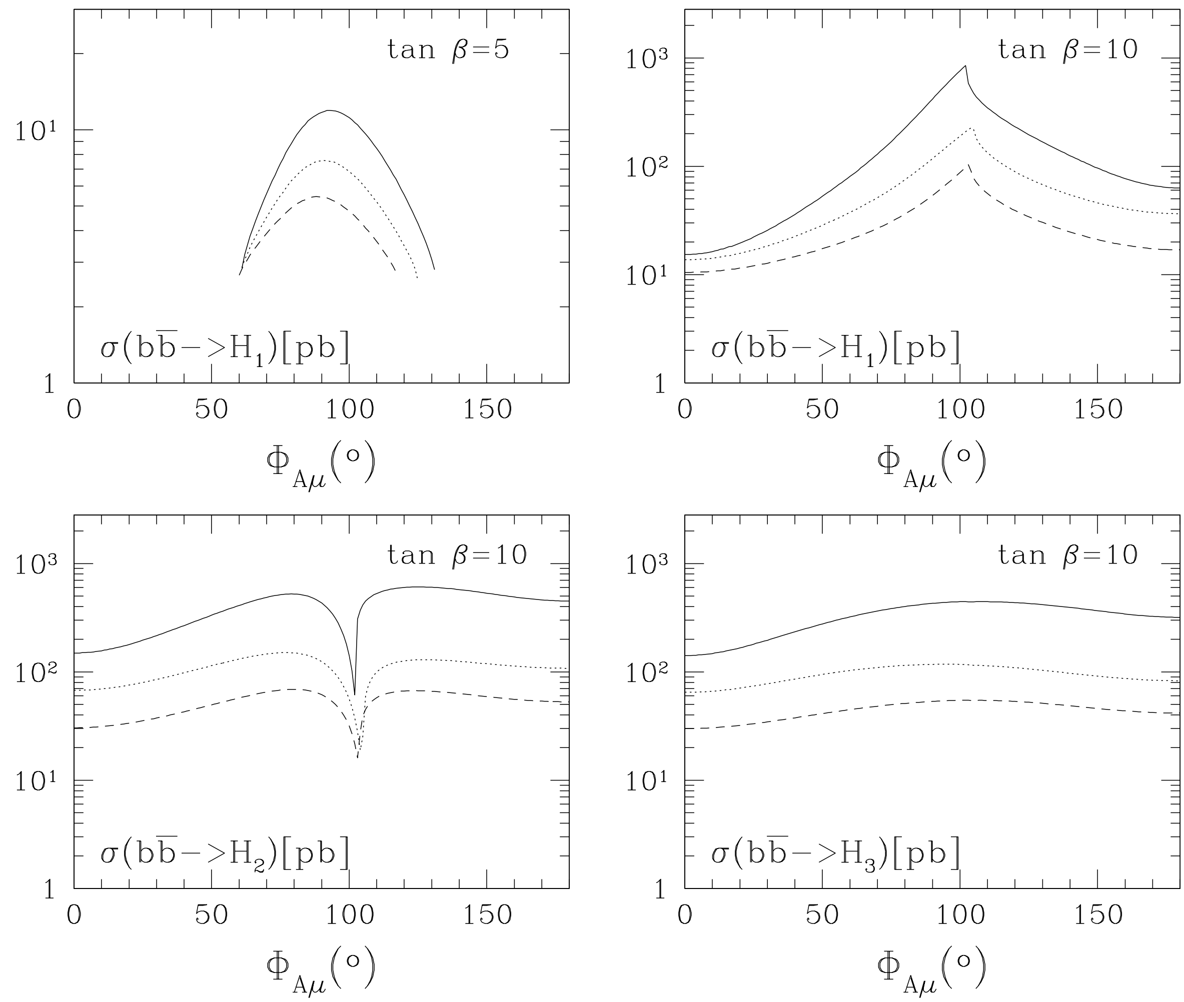

Figure 4: *Partonic cross sections for the b-quark fusion production of H_1, H_2, and H_3 vs. $\Phi_{A\mu}$, for $\Phi_{g\mu} = 0^{\rm o}$ (dashed lines) and $180^{\rm o}$ (solid lines). The supersymmetric spectrum is that used also for Fig. 2. In dotted lines are also the cross sections with no m_b-corrections.*

Moreover, at $\tan\beta = 5$ the eigenstate H_1 has still a large CP-even component.

Overall, the m_b corrections increase the cross sections $\sigma(b\bar{b} \to H_i)$ with respect to the uncorrected ones for $\Phi_{g\mu} = 180^{\rm o}$, decrease them for $\Phi_{g\mu} = 0^{\rm o}$. The impact of these corrections, is in general always large, producing enhancements up to one order of magnitude and suppressions down to -60%. Notice that the corrections remain surprisingly large for H_2 and H_3, also in the CP-conserving cases $\Phi_{g\mu} = 180^{\rm o}$, $\Phi_{A\mu} = 0^{\rm o}$ ($+120\%$); $\Phi_{g\mu} = 180^{\rm o}$, $\Phi_{A\mu} = 180^{\rm o}$ ($+300\%$); as well as $\Phi_{g\mu} = 0^{\rm o}$, $\Phi_{A\mu} = 0^{\rm o}, 180^{\rm o}$ (-50%). They are more modest for H_1 at $\Phi_{g\mu} = 0^{\rm o}, 180^{\rm o}$, $\Phi_{A\mu} = 0^{\rm o}$ (i.e. $\sim \pm 20\%$), but still $+70\%$ and -50% at $\Phi_{g\mu} = 0^{\rm o}, 180^{\rm o}$ and $\Phi_{A\mu} = 180^{\rm o}$.

After convoluting the parton distribution functions, we obtain the hadronic cross sections for the Tevatron ($\sqrt{s} = 1.96\,{\rm TeV}$) and the LHC ($\sqrt{s} = 14\,{\rm TeV}$). These are shown in Fig. 5 vs. $\Phi_{A\mu}$, for $\tan\beta = 10$ and two values of $\Phi_{g\mu}$: $\Phi_{g\mu} = 0^{\rm o}$ (dashed lines) and $\Phi_{g\mu} = 180^{\rm o}$ (solid lines). We have used the leading-order CTEQ6L [22] parton distribution functions and chosen

the factorization scale $Q = m_{H_i}/4$. This has been suggested in most of the papers in Ref. [23] as the scale that minimizes the next-to-leading-order QCD corrections to these cross sections when no threshold corrections to m_b are kept into account. Although this should be explicitly checked, we believe that the inclusion of these corrections, which amounts to substituting the tree-level Yukawa couplings with effective ones, should not affect substantially this result. We notice also that these supersymmetric threshold corrections capture the main part of all supersymmetric corrections to the production cross sections of neutral Higgs bosons through b-quark fusion. Other corrections, with a nontrivial dependence on the momenta of the H_i bosons are of decoupling nature, and therefore subleading. (See the explicit check for the related processes $gb \to bH_i$ in the CP-conserving case of Ref. [24].)

As already observed at the partonic level, all three production cross section can deviate considerably from those obtained in CP-conserving scenarios and the impact of the supersymmetric threshold corrections to m_b is very important. The production cross section for the lightest Higgs boson through the b-quark fusion is, in general, comparable to the production cross section via gluon fusion. For some values of $\Phi_{g\mu}$ and $\Phi_{A\mu}$, it can even be larger. For example, for $\Phi_{g\mu} = \Phi_{A\mu} \sim 100^{\rm o}$, a production cross section via gluon fusion of $\lesssim 30\,{\rm pb}$ at the LHC, which can be easily evinced from Ref. [20] for the same set of supersymmetric masses used here, in particular the same value of m_{H_1}, is indeed smaller than the corresponding cross sections in Fig. 5. (The analysis of Ref. [20] does not include the supersymmetric threshold corrections to m_b. Their impact, however, is expected to be less dramatic in the case of the gluon-fusion production.) Even for different values of $\Phi_{g\mu}$ and $\Phi_{A\mu}$, the b-quark fusion production mechanism cannot be easily dismissed as subleading. No comparison between the two production mechanisms for the two heavier Higgs bosons is possible at the moment, since in the existing studies [25] of H_2 and H_3 production via gluon fusion different supersymmetric spectra than those considered here are analyzed. For all three mass eigenstates H_i, however, the cross sections corresponding to the two production mechanisms have rather different dependences on the supersymmetric spectrum. The results shown here for the b-quark fusion cross sections, for example, remain unchanged for values of $M_{\rm SUSY}$ different from our representative value of $0.5\,{\rm TeV}$, provided the relative size of the different supersymmetric parameters is not changed. The same is not true for the gluon-fusion cross sections, which are sensitive to the absolute value of $M_{\rm SUSY}$.

More investigations, theoretical and experimental, are needed to unravel all implications of the enhanced cross sections presented here. A detail comparison of the yield of all neutral Higgs bosons of supersymmetric scenarios through b-quark fusion, as opposite to the yield through gluon fusion and through Higgs strahlung (which is relevant for the Tevatron), will have to be performed. Moreover, all subsequent decay modes for these Higgs bosons need to be studied to allow unambiguous interpretations of possible Higgs bosons signals, which hopefully will be detected at the Tevatron and/or the LHC. All future analyses need to be carried out at the same level of precision, i.e. incorporating the threshold corrections to m_b that, as proven in this paper, turn out to be very important in these scenarios, not only for large but also for intermediate values of $\tan\beta$.

To summarize, we have studied the effects of threshold corrections to the b-quark mass on the production of neutral Higgs bosons via b-quark fusion, $b\bar{b} \to H_i$, in supersymmetric

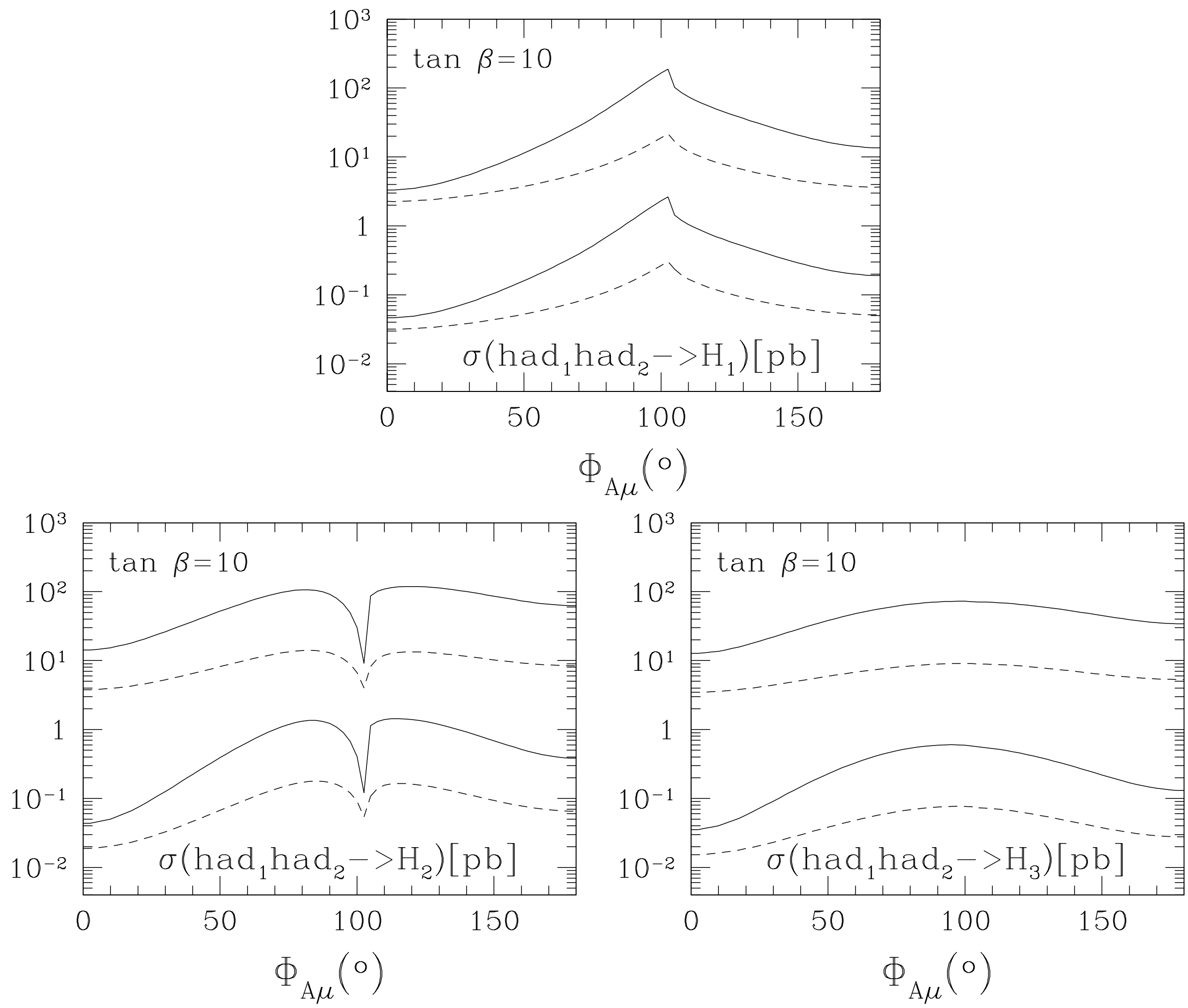

Figure 5: *Cross sections for the production of H_1, H_2, and H_3 vs. $\Phi_{A\mu}$, for $\Phi_{g\mu} = 180^{\rm o}$ (solid lines) and $0^{\rm o}$ (dashed lines), at the LHC with $\sqrt{s} = 14$ TeV (two upper lines) and at the Tevatron with $\sqrt{s} = 1.96$ TeV (two lower lines).* $\rm had_1 had_2$ *= pp for the LHC, $p\bar{p}$ for the Tevatron.*

scenarios with large CP-violation in the Higgs sector. This is assumed to be induced by explicit phases in supersymmetric and supersymmetry-breaking parameters. For these particular scenarios, we have found that:

i) large phases of the combination $M_{\tilde{g}}\mu$, i.e. $\Phi_{g\mu} = 180^{\rm o} \pm 30^{\rm o}$, for all values of $\Phi_{A\mu}$, can drive the mass squared of the lightest $\tilde{b}$ squark to negative values and, depending on the supersymmetric spectrum, the b-quark Yukawa coupling to nonperturbative ones;

ii) large deviations in the behaviour of the b-quark fusion cross sections, with respect to the same cross sections in CP-conserving scenarios, are obtained for $\Phi_{A\mu} \sim 100^{\rm o}$ when the Higgs mixing is maximal;

iii) the supersymmetric corrections to the b-quark fusion production cross sections are in general very large, even for the four CP-conserving scenarios obtained by fixing the values of $\Phi_{g\mu}$ and $\Phi_{A\mu}$ at $0^{\rm o}$ or $180^{\rm o}$. Among these four scenarios, those with $\Phi_{g\mu} = \Phi_{A\mu} = 180^{\rm o}$ give

the largest cross sections $\sigma(\mathrm{had}_1\mathrm{had}_2 \to b\bar{b} \to H_i)$ both at the Tevatron and at the LHC.

We conclude that in CP-violating supersymmetric scenarios, the production of neutral Higgs bosons through b-quark fusion cannot be neglected, in general, with respect to the production through gluon fusion. Dedicated theoretical and experimental studies should be carried out in order to allow a reliable detection of these mixed state neutral Higgs bosons, or to constrain the CP-violating scenarios that predict them.

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